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Myoung-jae LEE
Singapore Management University, mjlee@smu.edu.sg

Wen Juan LI
University of Tsukuba

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Drift and diffusion function specification for short-term interest rates

Myoung-jae Lee[a,*], Wen-juan Li[b]

[a]*School of Economics and Social Sciences, Singapore Management University, Singapore, 259756, Singapore*
[b]*Graduate School of Management Science, and Public Policy Studies, University of Tsukuba, Tsukuba, Ibaraki, Japan*

Abstract

Various stochastic differential equation models for short rates (r_t) have been proposed, where the change ($\Delta r_t = r_t - r_{t-1}$) is modeled as a sum of drift and diffusion terms depending on r_{t-1}. These models, however, have some shortcomings. First, the same model may not apply to all countries. Second, the drift and diffusion may depend not only on r_{t-1} but also on further lags. Third, not just the own lagged rates, but also other countries' rates may matter. These questions are empirically analyzed for six major countries with the following findings. First, there are considerable differences in drift and diffusion across the countries. Second, the drift and diffusion often depend on r_{t-2} (and r_{t-3}). Third, foreign rates exert substantial effects.

Keywords: Short rate; Diffusion; Spatial correlation

JEL classification: C51; G10

1. Introduction

Short-term interest rate r_t ("short rate") is one of the important prices in finance. Stochastic differential equation

$$\mathrm{d}r_t = \mu(r_t,\alpha)\mathrm{d}t + \sigma(r_t,\beta)\mathrm{d}w_t \tag{1.1}$$

* Corresponding author. Tel.: +65 68220856; fax: +65 68220833.
E-mail address: mjlee@smu.edu.sg (M. Lee).

has been used to describe its law of motion, where $\mu(r_t,\ \alpha)$ is the 'drift' function with parameter α, $\sigma(r_t,\ \beta)$ is the 'diffusion' function with parameter β, and w_t is a standard Brownian motion. One example is

$$\mu(r_t,\alpha) = \alpha_1 + \alpha_2 r_t, \quad \alpha \equiv (\alpha_1,\alpha_2)', \quad \sigma(r_t,\beta) = (\beta_1 r_t^{\beta_r})^{1/2}, \quad \text{and } \beta \equiv (\beta_1,\beta_r)'. \tag{1.2}$$

Chan et al. (1992) consider (1.2) nesting eight existing models to use GMM. Aït-Sahalia (1996a) derives an equation involving $\mu(r_t,\ \alpha)$, $\sigma(r_t,\ \beta)$, and density $\pi(r_t)$ of r_t. Substituting the LSE for $\mu(r_t,\ \alpha)=\alpha_1+\alpha_2 r_t$ and a nonparametric estimator for $\pi(r_t)$ into the equation, $\sigma(r_t,\ \beta)$ is estimated nonparametrically. Aït-Sahalia (1996b) derives an implied density for r_t under a parametric drift and diffusion; comparing the implied density with a nonparametric density estimator, the parametric specification is tested. Stanton (1997) estimates both drift and diffusion nonparametrically. Ahn and Gao (1999) consider

$$\mu(r_t,\alpha) = \alpha_1 + \alpha_2 r_t + \alpha_3 r_t^2 \quad \text{and } \sigma(r_t,\beta) = (\beta_1 + \beta_2 r_t + \beta_3 r_t^3)^{1/2}. \tag{1.3}$$

In the diffusion, r_t^3 appears instead of $r_t^{\beta_r}$ because the literature provides convincing evidences for $\beta_r \simeq 3$; another reason is that the identification of β_r is "fragile" when $\beta_1 \simeq 0$ in (1.2) (see Chan et al. (1992, p.1218) and Aït-Sahalia (1996b, p.412)).

Other than GMM and nonparametric methods, MLE is possible if (1.1) is solvable for r_t (Overbeck and Rydén (1997)). For all estimation methods other than Aït-Sahalia (1996a,b), discretization was used: for (1.3),

$$\Delta r_t = (\alpha_1 + \alpha_2 r_{t-1} + \alpha_3 r_{t-1}^2)\Delta t + (\beta_1 + \beta_2 r_{t-1} + \beta_3 r_{t-1}^3)^{1/2}\sqrt{\Delta t}u_t, \qquad u_t \sim N(0,1). \tag{1.4}$$

Absorbing Δt into β makes only a scale difference for β, and tests involving β are not affected; set Δt=1 from now on.

In this paper, we examine the specification for (1.1) focusing on two neglected aspects. First, instead of r_{t-1}, we allow *multiple lags* r_{t-1}, r_{t-2}, and r_{t-3}; e.g., discretization may result in relevance of r_{t-2} and r_{t-3} even when they are not relevant in continuous time. Second, for all we know, interest rates are *spatially correlated* with one country's rate affecting other countries' rates: six major countries' rates will be examined jointly.

With many explanatory variables owing to multiple lags and foreign rates, nonparametric methods are not attractive (see also Chapman and Pearson, 2000). Instead, we use extensive parametric models. For drift, r_{r-j}, r_{t-j}^2, and r_{t-j}^{-1} are used for j=1,2,3, along with the first lagged foreign rates; for diffusion, $r_{r\text{-}j}$, r_{t-j}^3, and r_{t-j}^{-1} are used for j=1,2,3, along with the first lagged foreign rates. This specification is more general than (1.4) and was motivated by that, in Aït-Sahalia (1996b),

$$\mu(r_t,\alpha) = \alpha_1 + \alpha_2 r_t + \alpha_3 r_t^2 + \alpha_4 r_t^{-1} \quad \text{and } \sigma(r_t,\beta) = (\beta_1 + \beta_2 r_t + \beta_3 r_t^{\beta_r})^{1/2} \tag{1.5}$$

is the only parametric specification not rejected. Our estimation takes two simple steps. First, the drift is estimated with LSE. Second, the diffusion is estimated with LSE using the squared residual as the dependent variable.

For the LSEs, the high multicollinearity problem makes the inferential process unstable. To get reliable standard errors, *overlapping block bootstrap* (Künsch, 1989) will be used with 500 replications. Block bootstrap of size l retains the temporal relations less than or equal to $l-1$ periods apart. For three lags, the minimal block length is l=4: we draw r_t, r_{t-1}, r_{t-2}, r_{t-3} at each draw. Künsch (1989) advises against drawing longer than the minimal length, but for comparison, we will present results for l=4,5. Lahiri (1999) compares various block-bootstraps (fixed-length and random-length, overlapping, and nonoverlapping) and recommends fixed-length overlapping blocks.

Our data are from the International Financial Statistics (an IMF publication). The sample contains 327 monthly observations from 1972 January to 1999 March. The following short rates are used: call money rate for France, Germany, and Japan; money market rate for Italy; overnight interbank rate for

Table 1
LSE for drift

	France	Germany	Italy	Japan	UK	USA
1	−0.847 (3.68)	−15.6 (3.46)	−0.403 (1.72)	0.050 (0.68)	0.367 (1.76)	−1.38 (1.63)
	(3.72)	(3.90)	(1.75)	(0.62)	(1.66)	(1.55)
r_{t-1}	.227 (2.39)	2.53 (3.59)	1.12 (2.59)		−0.354 (2.88)	.353 (2.70)
	(2.38)	(4.12)	(2.55)		(2.84)	(2.91)
r_{t-1}^2	−0.010 (3.65)	−0.131 (3.70)	−0.031 (1.98)	0.016 (3.50)		
	(3.78)	(4.34)	(1.95)	(3.33)		
r_{t-1}^{-1}		28.2 (3.22)		−0.49 (2.72)		
		(3.56)		(2.56)		
r_{t-2}	−0.223 (1.90)		−1.10 (2.51)		0.303 (2.51)	1.48 (2.26)
	(1.97)		(2.47)		(2.51)	(2.27)
r_{t-2}^2			0.027 (1.73)	−0.018 (3.84)		−0.070 (2.87)
			(1.71)	(3.66)		(2.91)
r_{t-2}^{-1}						21.19 (2.34)
						(2.31)
r_{t-3}						−1.60 (2.70)
						(2.65)
r_{t-3}^2						.059 (2.61)
						(2.59)
r_{t-3}^{-1}						−18.65 (2.16)
						(2.10)
r_{t-1_f}			0.051 (1.90)		0.117 (1.84)	
			(1.99)		(1.85)	
r_{t-1_g}	0.099 (3.32)				−0.107 (2.40)	
	(3.24)				(2.30)	
r_{t-1_i}	0.056 (2.17)			−0.022 (3.26)	−0.072 (2.21)	
	(2.06)			(3.08)	(2.27)	
r_{t-1_j}						
r_{t-1_uk}	−0.029 (2.49)					
	(2.65)					
r_{t-1_us}	0.085 (3.18)	0.058 (3.54)	0.044 (1.91)	0.038 (4.29)	0.076 (2.45)	
	(3.13)	(3.76)	(1.83)	(4.08)	(2.19)	
R_{adj}^2	0.198	0.310	0.214	0.192	0.132	0.302

UK; and federal funds rate for U.S.A. The spatial correlations hover around 0.5, ranging from 0.25 to 0.85.

2. Drift estimation

Interest rates have high temporal correlations (e.g., COR(r_t, r_{t-1})=0.98, COR(r_t, r_{t-2})=0.93, and COR(r_t, r_{t-3})=0.89 for U.S.A.), which causes a severe multicollinearity problem: with all variables in, no variable is significant. Hence, we eliminate some variables; for each country, we keep only (almost) significant variables. Table 1 shows the LSE for drift and the bootstrap absolute t-values. Two t-values are provided: the one for l=4 is next to the estimate, and the other for l=5 is below the l=4 t-value; overall, the two sets of t-values differ little. The second subscripts in r_{t-1_f}, r_{t-1_g}, r_{t-1_i}, r_{t-1_j}, r_{t-1_uk}, and r_{t-1_us} stand for the six countries.

First, the drift functions are nonlinear in its own lagged rates other than in UK. Second, each country needs a different model in terms of the lag length and functional form. Third, there are substantial spatial effects. Fourth, U.S.A. is not affected by any country, but U.S.A. affects all other countries. Fifth, Japan is influenced by U.S.A. and Italy, but does not affect any other country. While the influence of U.S.A. is plausible, that of Italy appears to be a statistical anomaly. Sixth, Germany is influenced only by U.S.A.

Table 2 presents $\partial\Delta r_t/\partial r_{t-1}$, $\partial\Delta r_t/\partial r_{t-2}$, and $\partial\Delta r_t/\partial r_{t-3}$. To be exact, e.g., the first term should be $\partial E(\Delta r_t|r_{t-1}, r_{t-2}, r_{t-3})/\partial r_{t-1}$, but we use $\partial\Delta r_t/\partial r_{t-1}$ to simplify notations. For example, France has $\partial\Delta r_t/\partial r_{t-1}$=0.23−0.02$r_{t-1}$, and this function is evaluated at the sample mean $\bar{r}_{t-1}$. Other than for UK, r_{t-1} has a positive effect and r_{t-2} has either a negative or almost zero effect. The effect of r_{t-2} is of comparable magnitude to that of r_{t-1}. For U.S.A. only, r_{t-3} is relevant with substantial effect. The last row shows the combined spatial effects $\Sigma_s \frac{\partial\Delta r_t}{\partial r_{t-1-s}}$. The magnitude is greater than that of r_{t-1} for France and smaller for other countries.

3. Diffusion estimation

Table 3 shows the LSE for diffusion and the bootstrap absolute t-values; the two sets of t-values for l=4,5 differ little. Table 3 repeats much the same lesson learned from drift. First, the diffusion is

Table 2
Effects on drift

	France	Germany	Italy	Japan	UK	USA
$\partial\Delta r_t/\partial r_{t-1}$	$0.23-0.02r_{t-1}$	$2.5-0.26r_{t-1}-28(1/r_{t-1}^2)$	$1.1-0.062r_{t-1}$	$0.032r_{t-1}+0.49(1/r_{t-1}^2)$	−0.35	0.35
at $\bar{r}_{t-1}$	0.051	0.16	0.36	0.18	−0.35	0.35
$\partial\Delta r_t/\partial r_{t-2}$	−0.22	0	$-1.1+0.054r_{t-2}$	$-0.036r_{t-2}$	0.30	$1.5-0.14r_{t-2}-21(1/r_{t-2}^2)$
at $\bar{r}_{t-2}$	−0.22	0	−0.43	−0.19	0.30	0.049
$\partial\Delta r_t/\partial r_{t-3}$	0	0	0	0	0	$-1.6+0.12r_{t-3}+19(1/r_{t-3}^2)$
at $\bar{r}_{t-3}$	0	0	0	0	0	−0.38
spatial	0.211	0.058	0.095	0.016	0.014	0

Table 3
LSE for diffusion

	France	Germany	Italy	Japan	UK	USA
1	−0.714 (1.64)	1.77 (2.58)	.512 (1.79)	−0.015 (0.35)	0.466 (1.01)	−0.345 (1.72)
	(1.64)	(2.44)	(1.67)	(0.36)	(0.98)	(1.80)
r_{t-1}			0.285 (2.14)	0.039 (5.04)		
			(2.29)	(5.02)		
r^3_{t-1}	0.00063 (1.30)					0.00052 (1.83)
	(1.37)					(1.87)
r^3_{t-2}	−0.00086 (1.92)					
	(2.14)					
r_{t-3}		0.292 (1.89)	−0.250 (2.05)			
		(1.88)	(2.16)			
r_{t-1_f}				−0.030 (2.43)		
				(2.70)		
r_{t-1_g}	0.151 (2.32)				−0.262 (2.59)	
	(2.34)				(2.72)	
r_{t-1_i}	0.082 (2.03)	−0.265 (2.93)		0.010 (1.43)		
	(2.07)	(2.87)		(1.57)		
r_{t-1_j}	−0.057 (1.68)	0.219 (2.53)	0.078 (2.11)			0.058 (2.49)
	(1.73)	(2.43)	(1.96)			(2.58)
r_{t-1_uk}	−0.042 (2.75)	−0.220 (3.14)		−0.009 (1.99)		
	(3.10)	(3.12)		(2.08)		
r_{t-1_us}		0.161 (2.39)	−0.125 (2.58)	0.017 (2.17)	0.326 (3.44)	
		(2.55)	(2.63)	(2.32)	(3.68)	
R^2_{adj}	0.154	0.151	0.065	0.158	0.039	0.160

sometimes nonlinear in lagged rates. Second, the diffusion varies across the countries. Third, r_{t-3} matters for Italy. Fourth, the spatial effects have nonnegligible magnitudes and the pattern differs from that for drift. For U.S.A., only r_{t-1} and the Japan effects are (almost) significant. For Japan, r_{t-1} and the effects from France, UK, and U.S.A. are significant. Also, Japan affects Germany, Italy, and U.S.A. For Germany, r_{t-3} is marginally significant and the spatial effects from most foreign countries are significant with fairly big magnitudes. For UK, there are only spatial effects from Germany and U.S.A.

Partial effect analysis on diffusion analogous to that for Table 2 is in Table 4. Again, similar results result: the effect magnitudes of r_{t-2} and r_{t-3} can be bigger than that of r_{t-1} as in France and Germany, and the spatial effect can be as large as that of r_{t-1} as France, Germany, and UK show.

Table 4
Effects on diffusion

	France	Germany	Italy	Japan	UK	USA
$\partial\Delta r_t/\partial r_{t-1}$	$0.0019r^2_{t-1}$		0.285	0.039		$0.0016r^2_{t-1}$
at $\bar{r}_{t-1}$	0.140		0.285			0.092
$\partial\Delta r_t/\partial r_{t-2}$	$-0.0026r^2_{t-2}$					
at $\bar{r}_{t-2}$	−0.210					
$\partial\Delta r_t/\partial r_{t-3}$		0.292	−0.250			
at $\bar{r}_{t-3}$		0.292	−0.250			
spatial	0.134	−0.105	−0.047	−0.012	0.064	0.058

4. Simulation study

The high degree of nonlinearity and multicollinearity posed difficulty for our procedure, the properties of which are explored with simulation here. The main problem in simulation was generating interest rates. Although the highly nonlinear models generated reasonable numbers for a while, eventually, interest rates became explosive/negative, because there is nothing built-in to prevent this. Hence, we set-up Design 1 as shown in Table 5.

First, we ran LSE or r_t on (1, r_{t-1}) using the U.S.A. data to get

$$\Delta r_t = 0.168 - 0.024 r_{t-1} = 0.024(7 - r_{t-1}); \tag{4.1}$$

this features mean-reversion to 7% and the '*speed of adjustment*' 0.024. Second, to allow for complex nonlinearity, we modified (4.1) to

$$\begin{aligned}\Delta r_t = \underset{\alpha_1}{0.168} - \underset{\alpha_{11}}{0.024}\, r_{t-1} - \underset{\alpha_{12}}{0.0024}\, r_{t-1}^2 + \underset{\alpha_{13}}{0.24}\, r_{t-1}^{-1} + \underset{\alpha_{21}}{0.024}\, r_{t-2} - \underset{\alpha_{22}}{0.0024}\, r_{t-2}^2 \\ + \underset{\alpha_{23}}{0.24}\, r_{t-2}^{-1} + \underset{\alpha_{31}}{0}\, r_{t-3} - \underset{\alpha_{32}}{0}\, r_{t-3}^2 + \underset{\alpha_{33}}{0}\, r_{t-3}^{-1} + \underset{\alpha_{sp}}{0.024}\, r_{f,t-1}\end{aligned} \tag{4.2}$$

where α_{sp}=0.024 (=$|\alpha_{11}|$) is for spatial effect. Since r_{t-1}^2 is roughly 7 times larger than r_{t-1}, we set α_{12} 10 times smaller than α_{11}; since r_{t-1}^{-1} is roughly 7 times smaller, we set α_{13} 10 times larger. It is hard to tell

Table 5
Monte Carlo study

Design 1: Parameter		T=300	T=600	Design 2:	T=300	T=600
		Est. (Reject)	Est. (Reject)	Parameter	Est. (Reject)	Est. (Reject)
α_1	0.168	−0.652 (0.03)	0.088 (0.02)	1	0.916 (0.06)	1.086 (0.30)
1+α_{11}	0.976	1.090 (0.58)	1.025 (0.94)	0.8	0.783 (0.55)	0.753 (0.91)
α_{12}	−0.0024	−0.011 (0.03)	−0.007 (0.02)	−0.02	−0.016 (0.10)	−0.017 (0.16)
α_{13}	0.24	0.501 (0.04)	0.478 (0.06)	2	1.920 (0.31)	1.897 (0.75)
α_{21}	−0.024	0.075 (0.01)	−0.024 (0.02)	−0.2	−0.236 (0.07)	−0.236 (0.24)
α_{22}	−0.0024	−0.012 (0.03)	−0.004 (0.02)	−0.02	−0.015 (0.12)	−0.016 (0.19)
α_{23}	0.24	0.723 (0.00)	0.244 (0.02)	2	2.005 (0.37)	1.971 (0.75)
α_{31}	0.000	−0.030 (0.01)	0.017 (0.03)	0	0.005 (0.03)	0.006 (0.03)
α_{32}	0.000	0.002 (0.03)	0.000 (0.03)	0	0.001 (0.03)	−0.001 (0.06)
α_{33}	0.000	0.007 (0.01)	0.030 (0.01)	0	0.126 (0.07)	0.022 (0.06)
α_{sp}	0.024	0.026 (0.13)	0.025 (0.28)	0.2	0.206 (1.00)	0.208 (1.00)
β_1	−1.180	1.449 (0.33)	1.334 (0.60)	1	1.103 (0.07)	0.927 (0.27)
β_{11}	0.060	−0.087 (0.04)	−0.082 (0.04)	0	−0.021 (0.02)	0.042 (0.07)
β_{12}	0.001	0.000 (0.01)	0.000 (0.05)	2	0.000 (0.04)	−0.001 (0.08)
β_{13}	3.647	−0.669 (0.06)	−0.390 (0.05)	0	−0.412 (0.00)	−0.170 (0.04)
β_{sp}	0.060	−0.055 (0.52)	−0.053 (0.72)	0.2	0.168 (0.59)	0.173 (0.81)

the first-lag dynamics from exactly the same second-lag dynamics in (4.2). Third, we ran the LSE of the squared residuals from (4.1) on (1, r_{t-1}, r_{t-1}^3, r_{t-1}^{-1}) to obtain

$$\hat{r}_t^2 = \underset{\beta_1}{-1.180} + \underset{\beta_{11}}{0.060}\, r_{t-1} + \underset{\beta_{12}}{0.001}\, r_{t-1}^3 + \underset{\beta_{13}}{3.647}\, r_{t-1}^{-1}(\underset{\beta_{\mathrm{sp}}}{0.060}\, r_{f,t-1}) \tag{4.3}$$

where β_{sp}=0.060 (=β_{11}) is for spatial effect; β_{sp} is not part of the LSE and it is added only to generate simulated data. Fourth, using (4.2) and (4.3) and the initial three-period U.S.A. rates, we generated T=300 (or 600) interest rates for two countries to estimate only one country model. The simulation repetition number is 200.

In Table 5, the median among the 200 estimates and the proportion of rejecting zero coefficient are shown. With T=300, the first-lag dynamics is better estimated than the second-lag dynamics for drift. For diffusion, the estimates have signs opposite to the true signs, but this is because the absolute value is taken on the diffusion estimates to prevent (negative)$^{1/2}$. The spatial effects are well picked up in both drift and diffusion, for the foreign rate does not suffer the multicollinearity problem. With T=600, the performance improves with lower biases and higher power. Still, the difficulty of separating the second-lag dynamics from the first remains. This is no surprise; as stated earlier, with all terms included in our real data analysis, no term was significant.

The multicollinearity problem can be avoided if the rates have more variation, but the variation due to the error term does not help. Instead, the variation from a higher speed of adjustment does. Estimating (4.1) with the German data, we get

$$\Delta r_t = 0.374 - 0.097 r_{t-1} = 0.097(3.86 - r_{t-1}); \tag{4.4}$$

the speed of adjustment 0.097 is faster than 0.024 in (4.1). We tried the German data but the outcome was little different from Design 1. In Design 2, we have Δr_t=0.2(5−r_{t-1}) where the speed of adjustment is twice the German speed and the stationary level is 5%. The remaining drift parameters are chosen following the same reason as for Design 1, and the diffusion parameters are chosen arbitrarily. The last two columns show that the first- and second-lag dynamics are well separated in the drift, but the diffusion parameters are not well estimated. As in Design 1, the spatial effects are easily picked up.

In summary, first, as the speed of adjustment increases, the dynamic feature due to different lags becomes easier to estimate. Second, the estimation procedure is conservative with low power. Third, spatial effects are well estimated whereas diffusion parameters are not.

5. Conclusion

For the drift and diffusion of a short rate r_t, we showed that r_{t-2} and r_{t-3} can be as relevant as r_{t-1}, that each country needs different drift and diffusion models, and that spatial effects can be as important as r_{t-1}.

Allowing for r_{t-2}, r_{t-3}, and foreign rates are not the only way to modify the short rate models; e.g., multifactor models may be used. A more important consideration could be allowing for big changes in foreign exchange markets and monetary policy regimes. Indeed, such changes took place during our data period: e.g., the U.S. Federal Reserve's money-targeting experiment during the Paul Volcker era;

speculative attacks against the exchange rates of France, Italy, and UK; and abandoning exchange rate pegs as in UK. Incorporating these, however, goes beyond the scope of this paper, and thus left for future research.

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